AL-BĪRŪNĪ ON THE DETERMINATION OF LATITUDES AND LONGITUDES IN INDIA

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Observations of al-Bīrūnī, an outstanding astronomical geographer of his time, on the Hindu methods of finding terrestrial latitudes and longitudes are of more than ordinary interest. His statement in his *India*: 'In what way the Hindus determine the latitude of a place has not come to our knowledge' is puzzling. Likewise, his remark: 'The Hindus employ in this subject methods which do not rest on the same principle as ours. They are totally different and however different they are, it is perfectly clear that none of them hits the right mark' requires closer scrutiny. This has been done in this paper, both by reference to Bīrūnī's own citations and to Sanskrit texts now available. Bīrūnī's own method of determining longitudes from latitudes and distances as given in his *Taḥdīd* and *al-Qānūn al-Mas'ūdī* is discussed.

Abū Rayḥān Muḥammad ibn Aḥmad al-Bīrūnī's scientific interests, as is well known, covered a wide range of subjects from astronomy, mathematics and related matters to medicine, religion, philosophy and magic. The encyclopaedic nature of his writings will be abundantly clear from Boilot's excellent bibliographic studies.[1] Kennedy has estimated that about 146 titles credited to Bīrūnī, of which only 22 works are at present known and 13 available in publication, could have easily run into 13,000 printed pages, a formidable output by any standard.[2] About fifty percent of this great bulk of writings concern astronomy, mathematics, astronomical geography and mechanics, which clearly emphasize his predilection for exact sciences,—subjects readily lending themselves to mathematical treatment. As an astronomer, he was highly competent, if not an innovator in the sense of proposing new planetary schemes or astronomical theories and concentrated his energies on comparative studies of different methods and theories in vogue among peoples of different cultures, both before and during his own times. This largely accounts for the great historical value of his writings in astronomy, and for that matter in other subjects in which the same attitude of the scientist-historian is discernible.

As an astronomical or mathematical geographer, Bīrūnī was outstanding,—a giant among giants, trying to do for medieval geography what Ptolemy had attempted for the ancient. While the great Alexandrian had to depend largely on travellers' and mariners' accounts and less on results of actual experimental observations because few were available, the Khwārazmain himself determined latitudes and longitudes of a large number of places with refined techniques and consequently with greater accuracy. While still in his teens, Bīrūnī determined the latitudes of Kāth by observing through a graduated ring the meridional altitudes of the sun. Later on he collaborated with Abū-l-Wafā (940-997/98), veteran astronomer of Baghdād, in arranging for simultaneous observations of a lunar eclipse (May 24, 997) and determined the longitude difference between Kāth and Baghdād. He made similar eclipse observations

from Gurgān (February 19 and August 14, 1003) and from Jurjāniyya (June 4, 1004) for the same purpose. As regards the much simpler determinations of latitudes from solar meridional altitudes observed by graduated rings, astrolabes or sun-dials, Bīrūnī carried out quite a number of them,—in fact, it appears that he did it as a matter of routine whenever he visited a new place, and utilized these results, in combination with known or measured terrestrial distances in *farasakhs*, to compute longitude differences by applying the methods of spherical triangles. His researches on the determination of longitudes and latitudes, the earth's dimension and many astronomical elements such as the obliquity of the ecliptic are set forth in his important work the *Kitāb fī* Tahdid Nihayat al-makin li-Tashih Masafat *al-Masākin*, simply the *Tahdīd*,[3] composed about 1025 A.D. some 25 years after his first major work the *Chronology of Ancient Nations* and six years before his most voluminous work on India, the *Ta'rīkh al-Hind*. These elements are also incorporated in his *Magnum opus al-Qānūn al-Mas'ūdī*.[4]

Latitudes

In his *Tarīkh al-Hind*, Bīrūnī maintains his interest in his astronomical geography emphasizing, as one would naturally expect of him, the Indian concepts and methods and comparing and contrasting them with those of the Arabs. On latitudes and their measurements from the equator north or south he notices the Hindu definition of the equator in chapter XXVI (The shape of heaven and earth according to the Hindu astronomers) as follows: "The line which divides the two earth-halves, the dry and the wet, from each other, is called *Niraksha*, i.e. *having no latitude* being identical with our equator. In four cardinal directions with relation to this line there are four great cities:—Yamakoṭi, in the east; Laṅkā, in the South; Romaka, in the west; Siddhapura, in the north"[5]. Furthermore:
"The earth is fastened on the two poles, and held by the axis. When the sun rises over the line which passes both through Meru and Laṅkā, that moment is noon to Yamakoṭi, midnight to the Greeks and evening to Siddhapura". This, he observes, was according to Puliśa and Āryabhaṭa.

Varāhamihira in his *Pañcasiddhāntikā* expresses the above as follows:[6]
udayo yo laṅkāyāṃ so'stamayah savitureva sidhapure |
Madhyāhno yamakoṭyāṃ romakaviṣaye' rddharātrah sah||

"What is sun-rise in Laṅkā is sunset in Siddhapura, midday in Yamakoṭi and midnight in Romaka country".

In the *Āryabhaṭīya*—where the same verse is met with, *yavakoṭyāṃ* is used for *yamakoṭyāṃ* and *syāt* for *sah*.[7]

Laṅkā lies on the equator and is central, the prime meridian passing through it, Yamakoti and Romaka lie on the east and west of Laṅkā, while Siddhapura is diametrically opposite to it. Bīrūnī tried to identify these places, but was baffled by the existence of Siddhapura 180° from Laṅkā where, according to the common and general belief, there should be 'nothing but unnavigable seas'.[8]

Bīrūnī reports the following values of latitudes of a few places he was able to collect from Hindu astronomical works in original Sanskrit or Arabic translations.[9]

Ujjain (Ujjayinī)	—	24°	(according to all Hindu canons)
		22°29′	(according to the *Composition of the Spheres* by Ya'qūb ibn Ṭāriq, based on *al-Arkand*, that is *Khaṇḍakhādyaka*).
		$4\frac{3}{5}$	(do)
Kanoj	—	26°35′	(Balabhadra)
Tāneshar (Thāneśwara)	—	30°12′	(Balabhadra)
		27° (?)	(Abū-Aḥmad)
Karli (?)	—	28°	(Abū-Aḥmad)
Kashmir	—	34°9′	(according to *Karaṇasāra* of Vitteśvara).

He rejects Yākūb's low value of $4\frac{3}{5}$° for Ujjayinī. 22°29′ is probably Āryabhaṭa's value which is actually 22°30′.[10] *Pañcasiddhāntikā* gives 24°.[11] Bīrūnī then gives his own findings of latitudes for a number of places as follows:

Lauhūr[(a)]	—	34°10′	Waihand (Attok)	—	34°30′
Ghazna	—	33°35′	Jailam	—	33°20′
Kābul	—	33°47′	Nandna[(d)]	—	32° 0′
Kandī[(b)]	—	33°55′	Sālkot (Sialkot)	—	32°58′
Dunpur[(c)]	—	34°20′	Mandakkakor[(e)]	—	31°50′
Lamghān	—	34°43′	Multān	—	29°40′
Purshāvar (Peshwar)	—	34°44′			

[(a) Different from Lahore. (b) On the road from Ghazna to Peshawar. (c) Unknown. (d) The fortress on the mountain Bālnāth overhanging the Jailam where Bīrūnī was kept under detention by Sultān Maḥmūd. (e) Fortress of Lahore. (See, Sachau's notes, *India*, II, 341)].

The main idea behind collecting the latitudes of a large number of places in India has been to compute their longitude differences from a knowledge of their measured distances which Bīrūnī had applied successfully as we shall see in what follows.

Bīrūnī has not discussed Hindu methods for determining latitudes, but what is very puzzling in his statement: "In what way the Hindus determine the latitude of a place has not come to our knowledge".[12] He had known the *Sindhind*, or the *Brāhmasphuṭasiddhānta*, the *al-Arkand* or the *Khaṇḍa-khādyaka*, both by Brahmagupta, in the Arabic translations of al-Fazārī and Ya'qūb ibn Ṭarīq, and the works of al-Khwārizmī, al-Kindī, Abū-Ma'shar and a few others and himself translated, from the original Sanskrit versions, the

Pauliśa Siddhānta and the *Khaṇḍakhādyaka*. In all these texts, the usual methods of determining latitudes from equinoctial shadows or from the sun's zenith distances and declinations on any day are discussed. Take, for example, the following rule given in the *Khaṇḍakhādyaka :* [13]

viṣuvatkarṇavibhakte saṅkucchāyāhate pṛthak trijye|
lambākṣajye cāpaṃ viṣuvajjyāyāḥ svadeśākṣaḥ||

"The gnomon and the (equinoctial) shadow multiplied separately by the radius and divided by the hypotenuse corresponding to the equinoctial shadow give the *R*sine of the colatitude and the *R*sine of the latitude of the place respectively. The arc corresponding to the *R*sine of the latitude is the latitude"

The method involves the measurement of the shadow cast by a gnomon of the sun at the meridian on the equinoctial day. The line joining the apex of the gnomon and the tip of the shadow is the hypotenuse. The angle contained between the gnomon and the hypotenuse is the latitude ϕ, and how $\sin\phi$ and therefore ϕ are to be found is clearly indicated.

The *Pañcasiddhāntikā*[14] gives the following rules which possibly formed part of the *Pauliśa Siddhānta :*

viṣuvaddinamadhyāhūcchāyāvargātsavedakṛtarūpāt|
mūlena śataṃ vimśa viṣuvacchāyāhataṃ chindyam||
labdhaṃ viṣuvajjīvā cāpamato'kso'tha vā yatheṣṭadine|
meṣādyapakramayutastulādiṣu vivarjitaḥ svākṣaḥ||

"The equinoctial shadow multiplied by 120 is to be divided by the square root of the sum of the square of the equinoctial midday shadow (of the gnomon) and 144. The quotient is the *R* sine of latitude and the corresponding arc the latitude. Alternatively, the sun's declination in the (six) signs beginning with Aries added to, and the same in the (six) signs beginning with Libra deducted from, the arc (determined by the above method) on any given day give the latitude of the place".

On the equinoctial day, the sun's zenith distance $z =$ the latitude ϕ of the place. On any other day, $\phi = z \pm \delta$ where δ is the sun's declination. According to the rule, ϕ and z are given by:

$$R \sin \phi = \frac{120\ S_E}{\sqrt{S^2_E + 144}}$$

$$R \sin \phi = \frac{120 S}{\sqrt{S^2 + 144}}$$

where S_E is the length of the equinoctial midday shadow, S the shadow length on any other given day and R the radius equal to 120′. 144 represents the square of the gnomon of 12 *aṅgulas* in height. Rules for finding the sun's declination on any day from its longitude and the greatest declination (24° assumed in *Pañcasiddhāntikā*) are given elsewhere in the text (III.35).

The *Pañcasiddhāntikā*[15] also contains rules for finding the latitude from the altitude of the Pole Star. A straight gnomon is to be inclined in such a

way that the observer's eye placed at its base, its top and the Pole Star all lie in a straight line. At Laṅkā this observation can be made with the gnomon lying flat on the ground ($\phi=0$) and at the Sumeru (North Pole) with the gnomon standing upright ($\phi=90°$). At intermediate regions, the gnomon will have to be inclined, and the perpendicular from its top to the ground will represent the sine of the latitude ($g\sin\phi$, where g=length of the gnomon) and the distance from the base of the gnomon to the foot of the perpendicular the sine of co-latitude ($g\sin(90-\phi)$). The rules are:

ṛjuśaṅkubudhnavinyastalocano nāmayettathā śaṅkum |
bhavati yathā śaṅkvagraṃ dhruvatārādṛṣṭimadhyastham ||
patitena bhavati vedho laṅkāyāmūrdhvagena tu sumerau |
vinatena cāntarāle phalakacchedārdha sūtrasame ||
tatrāvalambako yaḥ so'kṣjya tasya śaṅkuvivaraṃyat |
viṣuvadavalambako'sau yāmyottaradikprasiddhikaraḥ ||

The *Brāhmasphuṭasiddhānta* which Bīrūnī constantly referred to gives rules[16] for calculating the latitude of a place from the equinoctial midday shadow of the gnomon.

Longitudes

Bīrūnī's discussions on longitudes are given in his *India* partly in chapters 29 and 30 entitled 'Definition of the inhabitable Earth according to the Hindus' and 'On Laṅkā, or the Cupola of the Earth', and more fully in Chapter 31 entitled ' On that difference of various places which we call the difference of longitude'. The main points are (a) the choice of the prime meridian, (b) the Hindu methods of determining longitudes and their correctness, and (c) reference to the methods he had himself followed in longitude determinations.

Regarding the choice of the prime meridian, a good deal of confusion prevailed in Bīrūnī's time, and there was no general agreement as to which meridian was to be treated as such. 'The theory of the Western astronomers on this point', writes Bīrūnī, 'is a double one'.[17] Some adopted the line passing through the coast of the Atlantic Ocean as the prime meridian, whereas, according to others, the line passing through the Islands of the Happy Ones was the beginning of longitude. In his *Taḥdīd* where he discussed the point more fully, he informed us that the confusion had really started with the Greeks who sometimes calculated the longitudes from the Canaries Islands (same as Fortunate Islands) and sometimes from the most distant points on the Atlantic Coast.[18] Bīrūnī's own preference was for the meridian passing through Susul-Aqsa, the farthest point on North Africa. Measured from this line, the longitudes of some of the places determined by him are as follows: Iskandariya (Alexandria)—51°54′; Bagdād—70°; Shirāz—78°33′; Kirman—80°.[19] The corresponding values measured from the Fortunate Islands and generally given on the astrolabes are 61°54′, 80° 88° and 91°30′[20]. The longitude difference of these two prime meridians is about 10°. Apart from

inaccuracies in measurements, this lack of general agreement and the omission to correct for one prime meridian longitude values taken from different tables were no doubt responsible for much confusion.

The Hindu astronomers, on the other hand, consistently, adhered to the meridian of Ujjayinī as their reference line on the belief that the inhabitable world extended in longitude in the direction of east and west through 180 degrees and that this line was central, being 90 degrees each from the western and the eastern limits of the inhabitable land mass.[21] The prime meridian extending from the North Pole (Meru) to Laṅkā on the equator was taken to pass through Ujjayinī in Mālava, Rohitaka in the district of Multān, Kurukṣetra in the plain of Thāneśvara, the river Yamunā on which Mathurā is situated and the mountains of Himavant covered with everlasting snow.[22] It appears that the above description of the prime meridian was taken by Bīrūnī from the *Pauliśa-Siddhānta*. Bhaṭṭotpala, in his commentary on the *Khaṇḍa-khādyaka,* quotes the following line from Ācārya Puliśa[23] who teaches that longitude corrections for planatory positions are not required for Ujjayinī, Rohitaka, Kurukṣetra, the Yamunā, the Himanivāsa and the North Pole as the line passing through them is centrally situated (prime meridian).

ujjayinīrohitakakuruyamunāhimanivāsamerūṇām |
deśāntaraṃ na kāryaṃ tallekhāmadhyavartitatvāt ||

In Arabic translations Ujjayinī became Arin. It is well known that Hindu astronomical texts were available to Muslim astronomers and scholars at the beginning of the Arab intellectual movement. In view of the importance of reckoning time and ascertaining the co-ordinates of terrestrial places so that prayer times can be accurately fixed and faces can be turned towards the Ka'ba during prayers, as required in their religion, these astronomical texts were highly valued and methods given therein adopted for computational purposes. Al-Khwārizmī, in his Astronomical Tables[24], states that calculations for mean positions of planets were 'made for the locality of Arin'. 'If we are removed in longitude from this (place, namely Arin)', he wrote further on, 'then the distance between our place and the locality of Arin must be taken into account. Thus having established, how many degrees and perhaps minutes our place is distant from Arin, one hour is to be reckoned for each 15 degrees'. Following the Indians, the Arab astronomers, at least in the initial phase, adopted the meridian of Ujjayinī as the reference line for longitude estimation and regarded Laṅkā as the centre of the inhabitable world,—the cupola of the earth.[25] After the introduction of Ptolemy's *Almagest* and *Geography,* the Eastern Arabs preferred the Greek methods and replaced the meridian of Arin by that of the Fortunate Islands as the prime meridian. In Spain, however, Arin and Hindu astronomical methods continued to remain important. When the need arose for adopting a new prime meridian farther west of the Fortunate Islands, the matter was decided on the basis of Toledo's longitude of 61°30′ west of Arin and Arin's longitude of 90° east of the new prime meridian.[26]

Regarding the methods of determining longitudes and expressing them, Bīrūnī observes: "The Hindus employ in this subject methods which do not rest on the same principle as ours. They are totally different; and howsoever different they are, it is perfectly clear that none of them hits the right mark"[27] Let us examine how far his observations are correct. Apart from the difference in the choice of the prime meridian discussed above, the Hindus, Bīrūnī points out, express longitude differences in *yojanas,* that is in linear measure unlike Muslim astronomers who express them 'by equatorial times corresponding to the distance between the two meridians'. Such differences are immaterial because longitude differences can be expressed in times, degrees and distances as Bīrūnī himself states: 'It is all the same whether these *equinoctial times* whatsoever their number for each meridian may be, are reckoned as 360th parts of a circle, or as its 60th parts, so as to correspond to the *day-minutes,* or as *farsakh* or *yojana*.'[28] Moreover, Hindu astronomers also gave longitude differences in time in connection with their rules for finding them from a lunar eclipse, which Bīrūnī himself notices and remarks as a correct method of calculation.[29]

In support of his statement that the Hindus express longitude, called by them *deśāntara,* in *yojanas,* Bīrūnī refers to a rule for introducing necessary corrections to the mean positions of planets when observed from a place east or west of Ujjayinī. Bīrūnī says: 'Further, they multiply the *deśāntara* by the mean daily motion of the *planet* (the sun), and divide the product by 4800. Then the quotient represents that amount of the motion of the star which corresponds to the number of *yojanas* in question, i.e. that which must be added to the mean place of the sun as it has been found for moon[30] or midnight of Ujain, if you want to find the longitude of the place in question.' This is Brahmagupta's rule given in the *Khaṇḍakhādyaka*[31] and quoted below:

> *ujjayinīyāmyottararekhāyāḥ prāgṛṇṃ dhanaṃ paścāt* |
> *deśāntarabhuktivadhāt khakhāṣṭavedaiḥ kalādyāptam* ||

Let the longitude difference of the place of observation = l *yojanas;*
the mean daily motion of the planet, *bhukti* $= v$ minutes (*kalās*);
circumference of the earth = 4800 *yojanas.*
The *deśāntara* correction λ' is given by

$$\lambda' = \frac{lv}{4800} \text{ minutes}$$

which is to be subtracted from, or added to, the calculated longitude of the planet for the meridian of *Ujjayinī* according as the observer's station is east or west of the prime meridian. This is a correct formula. The last part of Bīrūnī's statement, 'if you want to find the longitude of the place in question' is either a mistranslation or a misreading of the text, for, by the above method the longitude of the planet for the place in question and *not the longitude of the place in question* is sought to be found.

In the subsequent discussion, Bīrūnī, however, rightly points out that in the rule of the *Khaṇḍakhādyaka* given above, it is not stated whether 4800 *yojanas* represent the circumference of the earth at the equator or that of the latitude circle of Ujjayinī. In this *Uttara Khaṇḍakhādyaka*, Brahmagupta gives a rule[32] for finding the so called corrected circumference of the earth, i.e. of the circle of latitude; it directs the *jyā* of the colatitude to be multiplied by 5000 and divided by the radius. That is,

$$\odot_{\phi} = \frac{5000\ R\ \sin(90-\phi)}{R} = 5000 \cos \phi$$

Clearly, 5000 *yojanas* represent the measure of the earth's circumference at the equator, which agree with Brahmagupta's diameter of 1581 *yojanas*. Does the figure of 4800 *yojanas* then represent the correct circumference at the latitude of Ujjayinī, 'as people are frequently misled to think'? To test this, the value of ϕ can be calculated from the above formula by substituting 4800 for $\odot_{\phi}$. Bīrūnī did this and obtained the value of $16\frac{1}{4}°$ for Ujjayinī whereas, the generally accepted value is 24°[33] (modern value 23° 11′ 6″).

Bīrūnī refers to a method due to *Karaṇatilaka*, according to which the diameter of the earth in multiplied by 12 and the product divided by the equinoctial shadow of the place.[34] That is, the diameter of the latitude circle of the place, is given by:

$$D_{\phi} = \frac{12\ D}{S}, \text{ where } D = \text{diameter of the equator.}$$

This is not correct because, as Bīrūnī explains,

$$\frac{12}{S} = \frac{R \cos \phi}{R \sin \phi} = \cot \phi$$

and not equal to $\cos\phi$ as the author of the *Karaṇatilaka* erroneously supposes it to be.

Vijayanandin's *Karaṇatilaka* is now available and Rizvi has given a translation of it, with commentaries and notes. Rizvi's translation runs as follows: "The diameter of the earth is 1600 *yojanas* and accordingly its circumference is 5028 *yojanas*. So multiply the circumference of the earth by 12 and divide the product by the hypotenuse of *palabhā;* the quotient will be the corrected circumference".[35] In the manuscript Rizvi worked with, 'corrected diameter' is mentioned, which he has amended to 'corrected circumference'. This is immaterial. The important difference is that, whereas this text, if properly translated, correctly directs the gnomon length to be divided by the hypotenuse of the equinoctial shadow to obtain $\cos\phi$, Bīrūnī in his *Indica* clearly states that it is to be divided by the equinoctial shadow. Which one is correct?

The longitude difference between two places used to be determined in ancient and medieval times in two ways; (a) from the time difference of an eclipse observed from the two places concerned, and (b) from the latitude differences of and the linear distance between two places. Bīrūnī notices both the methods in Indian astronomical texts. The former method consisted in finding the difference, in day-minutes, between the time of appearance of a lunar eclipse in the two places and converting the time difference into *yojanas* by multiplying it by the circumference of the earth and dividing the product by 60.[36] The rational is that a time-difference of 60 minutes or *ghaṭikās* or *nāḍikās* (=1 day) corresponds to the circumference of the earth in *yojanas*. The rule given in the *Brāhmasphuṭa-siddhānta*[37] runs as follows (only a part is quoted);

pragrahanāntaraghaṭikābhūparidhihatā vibhājayet ṣaṣṭyā |

Brahmagupta's scholiast Pṛthūdakasvāmin, in his commentary on the *Khaṇḍa-khādyaka,* while explaining rule 15 of the first chapter concerning the longitude correction of the mean position of the planets, explains the above method of determining the longitude difference in minutes between two places by the time-difference of a lunar eclipse and expressing the same in *yojanas*.[38] He illustrates the rule in the case of Kurukṣetra where the eclipse is observed $1\frac{1}{2}$ *ghaṭikās* after the calculated time for the meridian of Ujjayinī. The longitude difference between Kurukṣetra and Ujjayinī therefore works out to $(3 \times 4800)/(2 \times 60)$ or 120 *yojanas*. The figure 4800 represents the earth's circumference in *yojanas* and not any corrected circumference. It is further to be noted that Pṛthūda-kasvāmin did not place Kurukṣetra on the meridian of Ujjayinī, but 120 *yojanas* east of it, to which Bīrūnī makes a reference.[39]

Regarding the second method of computing longitudes from latitudes. Bīrūnī refers to a method by al-Fazārī taken from some Hindu work, in which longitudes are sought to be calculated from latitudes alone, and very rightly observes that 'it is impossible to determine the distance between two places and the difference of longitude between them by means of their latitudes alone.[40] Bīrūnī then referred to another Hindu method 'based on the same principle', of which the inventor was not known. The method is: "Multiply the *yojanas* of the distance between two places by 9, and divide the product by (lacuna); the root of the difference between its square and the square of the difference of the two latitudes. Divide this number by 6. Then you get as quotient the number of day-minutes of the difference of the two longitudes" Compare the above method with the following rule of the *Pañcasiddhāntikā*, ascribed to Puliśa:[41]

trikṛtighnāt khavasuhṛtādyojanapiṇḍātsvatāditajjahyāt |
akṣadvayavivarakṛtiṃ mūlāḥ ṣaṭkoddhṛtā nāḍyaḥ ||

"Multiply (the distance between two places in) *yojanas* by 3^2 and divide by 80 and take the square (of the quotient); deduct from it the square of the difference of the two latitudes; the square root (of the remainder) divided by 6 gives (the longitude difference) in *nāḍis*."

The lacuna mentioned in Bīrūnī's reference is 80. Now 9/80 equals 360/3200 and represents the angular distance in degrees corresponding to a *yojana* on the surface of the earth whose circumference is assumed to be 3200 *yojanas*. This is also clearly explained in the *Pañcasiddhāntikā* in XII, 15. The first part of the rule seeks to convert the linear distance between two places into an angular measure. The rational of the second part of the rule is to assume a right-angled triangle formed between the latitude difference AB $(=\phi_2-\phi_1)$, the longitude difference BC and the difference AC between the two places, which is the hypotenuse (Fig. 1).

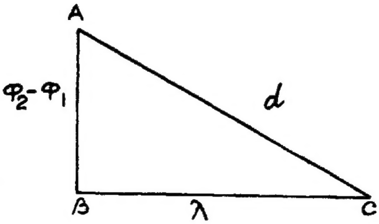

FIG. 1

This method cannot obviously be the same as that of al-Fazārī because it uses both the linear distance and the latitude difference. The method is no doubt defective in its simplified assumption where working with spherical triangles is involved, and is inferior to the procedure Bīrūnī followed in his own work on longitude measurements and explained below. But it must be stated in passing that Hindu astronomers were quite conscious of the limitations of this method. Bhāskara I, in his *Mahābhāskarīya,* for example, gives the same method as found in the *Pañcasiddhāntikā,* criticizing the rule at the same time as follows: "The distance (obtained above) has been stated to be incorrect by the disciples of (Ārya)bhaṭa, who are well versed in astronomy, on the ground that the method of knowing the hypotenuse is gross. (Those) wise people further say that on account of the sphericity of the earth (also), the method used for deriving the above rule commencing with *akṣa* is inaccurate".[42] Bhāskara I then discusses the eclipse method of finding longitudes and remarks that it is capable of yielding more accurate values.

The *Pañcasiddhāntikā* gives the longitude difference, in *nāḍikās,* between Alexandria and Ujjayinī as $7\frac{1}{3}$ and between Alexandria and Banaras as 9,[43] presumbly determined by the above mentioned approximate method. The differences work out to 44° and 54° respectively, which agree better with those based on modern longitude values than on the medieval, as the following table shows.

Table 1

	Pañca-siddhāntikā	Astrolabe	(medieval)[a]	Modern	
	Longitude from Alexandria	Longitude from Fortunate Island	Longitude from Alexandria	Long. from Greenwich	Long. from Alexandria
Alexandria	0	61°54′	0	29°51′	0
Ujjayinī	44°	102° 0	40° 6′	75°47′	45°56′
Banaras	54°	117°20	55°26′	83° 0	53° 9′

[a] Taken from Kaye, *Astronomical Observatories of Jai Singh*, pp. 128-29.

I shall conclude this paper by giving Bīrūnī's own method of computing longitude from latitudes and distances between two places as explained by Schoy.[4] Bīrūnī considered this method superior to that based on eclipses, 'because the first appearance and the end of the visibility of the eclipse, which are its most critical moments, can only be observed approximately'.[44]

Let the longitude difference between Shirāz and Baghdād be determined from the following data:

The latitude of Baghdād, $\phi_1 = 33°\ 25'$

The latitude of Shirāz, $\phi_2 = 29°\ 36'$

The distance between Baghdād and Shirāz
(after corrections for straightening), $d = 153$ *farsangs* $= 8°6'$.

Bīrūnī adopted 6800 *farsangs* for the circumference of the earth, making 1 *farsang* equal $0°\ 3'\ 10.58''$[45].

In Fig. 2, N is the North Pole; NBTP is the meridian of Baghdād B and NRSQ that of Shirāz S; BR and TS are segments of parallels of latitude through B and S respectively.

Arc $BP = \phi_1$
Arc $SQ = \phi_2$
Arc $BS = d$

It is required to find the arc PQ on the equator, which is the longitude difference between B and S.

The rectilinear figure obtained by joining B R S T is a cyclic trapezium, of which BR is parallel to TS, BT and RS are equal and the two diagonals are also equal. According to Ptolemy's Theorem,

$$BS.\ TR = BR.\ TS + BT.\ RS \quad \ldots\ldots(1)$$

or, $BS^2 - BT^2 = BR.\ TS$

$$\text{or,}\quad \frac{BS^2 - BT^2}{BR^2} = \frac{TS}{BR} \qquad \ldots.(2)$$

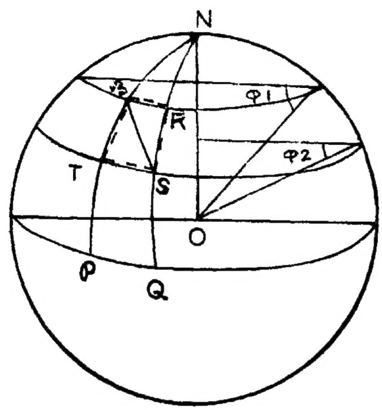

Fig. 2

$$\text{Now,}\quad \frac{TS}{BR} = \frac{\text{lat. circle through S}}{\text{lat. circle through B}} = \frac{2\pi R \cos \phi_2}{2\pi R \cos \phi_1} = \frac{\cos \phi_2}{\cos \phi_1} \qquad .. \quad (3)$$

$$\text{Likewise,}\quad \frac{BR}{PQ} = \frac{2\pi R \cos \phi_1}{2\pi R} = \cos \phi_1 \qquad .. \quad (4)$$

Substituting (3) and (4) in (2),

$$\frac{BS^2 - BT^2}{PQ^2 \cos^2 \phi_1} = \frac{\cos \phi_2}{\cos \phi_1}$$

$$PQ = \sqrt{\frac{BS^2 - BT^2}{\cos\phi_2 \cos\phi_1}} \qquad .. \quad (5)$$

By converting the arcs into chords with the help of his sine and cosine tables, Bīrūnī obtained the following values for his chords BS and BT and the cosine functions in sexagesimal units:

$$\begin{array}{llllll} BS & = & 0 & 8' & 28'' & 32''' \\ BT & = & 0 & 3' & 59'' & 46''' \\ \cos\phi_1 & = & 0 & 50' & 4'' & 12''' \\ \cos\phi_2 & = & 0 & 52' & 10'' & 17''' \end{array}$$

From (5), the value of the chord PQ works out to 0 8′ 17″ 16‴ and that of the arc PQ, that is, the longitude difference between Baghdād and Shirāz, to 8°33′ 32″[46].

This difference, as per modern longitude values of these two places, is 8° 16′, which indicates the degree of accuracy attained by Bīrūnī in his measurements.

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7 *Aryabhaṭī, Gola,* 13.

8 *India,* I, 303-4; 308-9.

9 *India,* I, 316-17.

10 *Aryabhaṭīya, Gola,* 14.

11 *Pañcasiddhāntikā,* 13, 19. The latitude of Ujjayinī (Avantī), is given as 213 *yojans,* which when converted into degrees by the rule 1° = 80/9 *yojanas* (13, 15), makes 24°.

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42 *Mahābhāskarīya,* ed. and translated by Kripa Shankar Shukla, Lucknow, 1960; II, 3, 4, 5. Rules 3, 4 give the method and rule 5 the criticism. Shukla's translation has been given in the citation.

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45 Kramers, *loc. cit.,* p. 187

46 Kramers checked up every step of the calculation and noticed discrepancies in the conversions from arcs into chords, which are not considerable, *loc. cit.,* pp. 187-88.